USING SONIFICATION TO DETECT WEAK CROSS-CORRELATIONS IN COUPLED EXCITABLE SYSTEMS

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ABSTRACT

We study cross-correlations in irregularly spiking systems. A single system displays spiking sequences that resemble a stochastic (Poisson) process. Linear coupling between two systems leaves the inter-spike interval distribution qualitatively unchanged but induces cross-correlations between the units. For strong coupling this leads to synchronization as expected but for weak coupling, both a good statistic and sonification reveal the presence of "motifs", preferred short firing sequences which are due to the deterministic spiking mechanism. We argue that the use of sonification for time series analysis is superior in the case where intrinsic non-stationarity of an experiment cannot be ruled out.

1. INTRODUCTION

The spiking of sensory and cortical neurons is highly irregular in general. Numerous investigations have been dedicated to the analysis of these firing patterns applying methods from univariate time series [1]. Recently, however, multivariate recordings become readily available and there is increased interest in multivariate time series analysis, i.e. the characterization of the relationship between spike trains. For example, in the context of auditory information processing the degree of coincidence between spike trains leaving the cochlea was found to take place in the cochlear nucleus [2] and was proposed to contain relevant information [3]. However, while perfectly or strongly synchronized time series are rather easy to characterize, this is not the case for weakly correlated time series. In particular, if the rate of spike coincidences is near the level expected for a random process, other means than synchronization analysis are required to distinguish weakly correlated from uncorrelated spike trains. Here we show in an explicit example that the search for preferred intervals *between* spike trains may provide such additional information and suggest that sonification provides an efficient medium to detect these intervals.

2. THE MODEL

We use a system of two chaotically firing oscillators with linear reversible coupling. The system composed of extended FitzHugh-Nagumo (FHN) models is given by:

$$
\begin{aligned}
\frac{dX_1}{dt} &= X_1(a-X_1)(X_1-1)-Y_1+I+dZ_1+D(X_2-X_1)\\
\frac{dY_1}{dt} &= b(X_1-Y_1)\\
\frac{dZ_1}{dt} &= \varepsilon - cX_1Z_1/(0.1+Z_1)\\
\frac{dX_2}{dt} &= X_2(a-X_2)(X_2-1)-Y_2+I+dZ_2+D(X_1-X_2)\\
\frac{dY_2}{dt} &= b(X_2-Y_2)\\
\frac{dZ_2}{dt} &= \varepsilon - cX_2Z_2/(0.1+Z_2)
\end{aligned}
\tag{1}
$$

with a=0.1, b=0.01, c=1.2, I=0.064, d=0.16, and ε=0.0001. D is the coupling constant and the only parameter varied in the present study.

3. NUMERICAL RESULTS

With the given set of parameters a single independent model exhibits chaotic self-excitation for D=0. This behavior is created by starting with the original FHN oscillator given by variables X and Y for d=0. As the FHN oscillator contains the harmonic oscillator it can be extended to generate a chaotic attractor in analogy with the Rössler equation [4]. To achieve this, we added a nonlinear switching variable Z and used the linear feedback controlled by parameter d to complete the three-variable autonomous chaotic system. A new property of the present system (compared to the standard type of chaos as e.g. in the Rössler equation) is that for some sets of parameters (e.g., I=0.062, other parameters as above) it is excitable: a short suprathreshold perturbation of one of its variables leads to a prominent spike (see [5] for an example and for a discussion of excitable chaos). The amplitude of this spike (for example in variable X) is large compared to the basal chaotic oscillations of the unperturbed system. After a single spike the value of variable X returns to the basal oscillations. Adjusting bifurcation parameter I to the value given above, the model exhibits chaotic self-excitation: spikes with their characteristic nonlinear wave-form arise spontaneously from the near-harmonic basal chaotic oscillations. Fig. 1 shows a time series in this regime. Dynamically the behavior is similar to the spontaneous spiking in the complex kinetic model in [5,6] where self-exciting chaos was introduced.

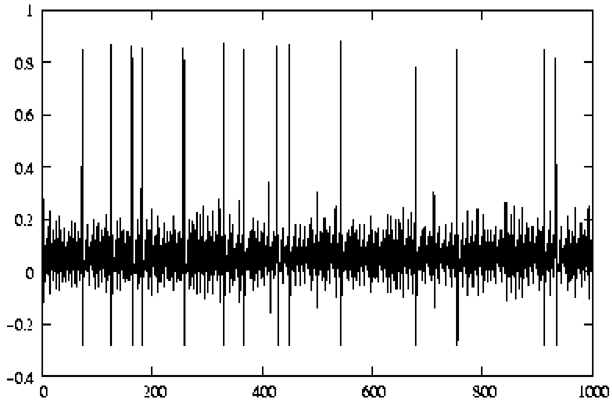

Figure 1. *Time series of variable X of a single oscillator as in eq. (1) with D=0. Other parameters as given in the text.*

Fig. 2 shows the normalized probability distribution of time intervals P(s) between successive spikes in a single extended FHN model (i.e., eq. (1) with D=0) in the parameter region of self-exciting chaos, and of two weakly coupled units (D=0.00125). (We call the coupling D weak when it assumes values that are less than 10% of the smallest value for which complete synchronization is found, in our case about 0.017.) For a broad range of next neighbor intervals we find a linear relationship for P(s) as a function of the normalised s in a semilogarithmic plot for both values of D. This is equivalent with an exponential decay of the distribution. As expected for an excitable system there is an absolute refractory period that leads to a gap with no events for short intervals. The slopes in the linear region of the distribution differ but this difference is merely due to a change in mean spike frequency: at D=0.00125 the average firing rate is lower than in the uncoupled case.

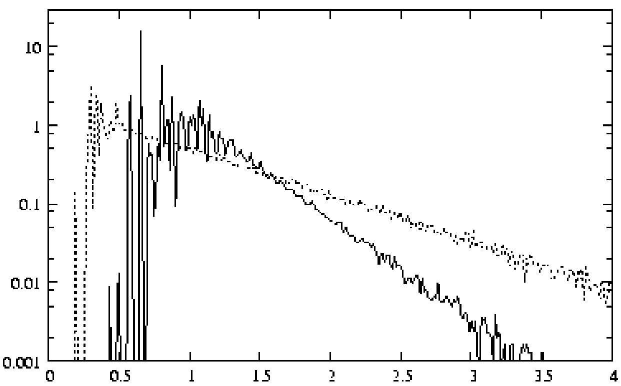

Figure 2. *P(s) distribution of inter-spike intervals in eq. (1) for D=0 (line) and D=0.00125 (dotted). 130000 spikes evaluated.(The x-axis is scaled on the average disctance between 2 successive spikes).*

The most significant deviation from a Poisson distribution with absolute refractory period stems from the sharp peaks to the left of the maximum of P(s). These maxima for short inter-spike intervals can be explained by the finite autocorrelation (or equivalently, a finite positive Lyapunov characteristic exponent) found in a chaotic system. Nearby trajectories of two such systems evolve similarly for short time scales and if two consecutive spikes occur within a short period (between one third and half of the average interval) they are bound to show this autocorrelation. In a sense, the spikes amplify these intrinsic short-term autocorrelations. Thus, at short time scales the deterministic origin of the spike sequence is recognizable in the deviation of the distribution from a Poisson behavior. However, the important observation for the present context is that this deviation from Poisson-type behavior is found with both values of the coupling constant and thus the distribution, apart from the change of mean firing rate, does not offer information about whether the two units are coupled or not.

To detect the effect of coupling it is advisable to use a measure that characterizes spike distances in distinct units (rather than in the same unit as in Fig. 2). We therefore evaluate the distribution of intervals *between* units. Fig. 3 shows a zoom of the probability distribution of the distances between spikes of *different* time series for D=0 and D=0.00125. For every spike in one unit the time interval to the next-nearest spike in the other unit was measured. The time interval Δt=0 was included and thus the height of the first bin at s=0 indicates the probability density of synchronized events. Clearly the number of coincidences of spikes is greater with finite D: coupling increases the probability of synchronized spiking. However, with the weak coupling chosen the synchronization is only partial. Most spikes do not coincide and visually the time series for the two cases cannot be distinguished in this respect.

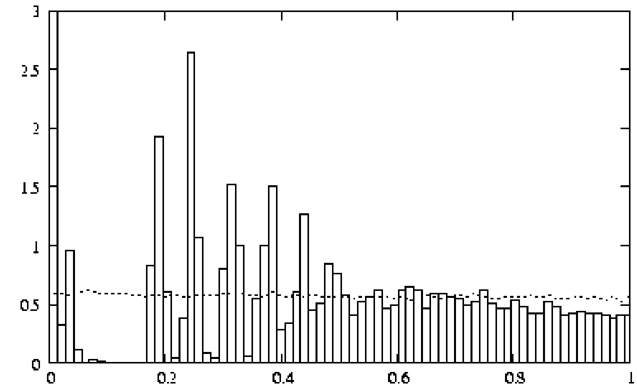

Figure 3. *Nearest neighbor spike interval distribution between the two units in eq. (1) for D=0 (dotted line) and D=0.00125 (bins). 130000 spikes evaluated.The first bin reaches 6.2 on the y-axis. (The x-axis is scaled on the average distance between 2 neighboring spikes in different units).*

The most obvious difference in the results for the two cases displayed in Fig. 3 is found for 0<s<0.5, i.e. for spike distances that are less than half of the average "inter-unit spike distance". For D=0 there is a plateau of nearly equal probability for 0<s<1 which is the expected result for two independent Poisson processes (the plateau being the consequence of the refractory period that has to be included). For D=0.00125 there is a gap of zero probability for 0.04<s<0.017. That is, given a spike in one unit, there is a minimum interval for which no spike will occur in the other unit. Following this gap there is a range of s with preferred intervals ($0.18<s<0.5$). For $s>0.5$ the distribution approaches the distribution of a stochastic process and almost no deviations are seen for larger spike intervals between units. To conclude this analysis we can say that weak coupling induces clear signatures of determinism if short inter-unit spike distances across units are evaluated.

Problems arise when the available time series are not as long as those in the above simulations. Evaluation of short time series leads to a significant blurring of the distributions and if one calculates the variance of the P(s) values it becomes clear that one can no longer distinguish easily between the two cases by this type of analysis. Both the homogeneous distribution at D=0 and the gap at D=0.00125 are corrupted. The statistics requires a large number of events to clearly show the differences.

4. SONIFICATIONS

At this point we turn to sonification of the time series as a means to detect the cross-correlations induced by finite coupling. The ear is particularly suited to detect rhythms and rhythmical changes in acoustic signals. Since the given data are time series that show structure along the time axis it is an obvious strategy to maintain the time stamp as sonification time.

However, the chosen temporal compression has a significant influence on the perceived structure and we therefore start with examples using **audification** to demonstrate the useful range of compressions. With high compression factors, spikes merge similar to granular synthesis to an acoustic texture, and features like roughness or timbre changes emerge. Dominant inter spike intervals can be perceived as pitch cues. This can be heard in the audification examples (all examples are to be found on our website [7]) S1c, rendered for compression factor 50. At lower time compression (example S1b, factor 6, S1a, factor 2), rhythm is perceived as the main structure and rhythmical changes can be followed. With even lower compression factors the time resolution would even allow to perceive phase differences, but this can't be heard in audification but in the amplitude modulation examples described next. Here we are particularly interested in the middle compression range of rhythmical structures.

Amplitude Modulation: A fundamental sonification idea is to simply use the amplitude of variable X to modulate the intensity of a given stationary sound. Thus spikes describe the amplitude envelope. Examples for this amplitude modulation technique are available for rates 0.1, 0.2 and 1.0 as examples S2a-c on our website. It can be heard that the small-amplitude oscillations between spikes cause a clear rhythm, and this obscures perceptually the spike rhythms between time series.

Event-based Rhythms: For this reason, and, since the major information lies in the accurate time point of spikes, we first extract the exact time when the signals exceed a threshold, X=0.65, and schedule acoustic events at the corresponding (mapped) time. To facilitate the perception of rhythms in the two units of eq. (1), we assign them a different pitch, and also route them to different audio channels. Examples in Sound S3 present such sonifications for the uncoupled and weakly coupled system. This event-based approach allows us to use more percussive sounds than for the original spike signals which have a non-zero transient. This assists the perceptual detection of rhythm.

In addition, such an approach enables us to use acoustic features of the events to convey detailed information about *local* properties like inter spike time intervals, etc.

Timbre Mapping: Specifically, we use additive synthesis with energy distributed on N harmonics for the events and - as a first example - use the intra-spike distance (time until the other time series spikes) to determine N for every spike. The larger this time, the more brilliant the sound. Thus rhythmical structuring also induces timbral structures. Sound examples S4 are provided on the website for different coupling constants and compression factors.

Non-stationary Time Series: Finally, as an important application for the analysis of experimental time series, we sonify the dynamics of the system with temporal variation of the coupling constant. This introduces non-stationarity to the spike pattern. Since the ear is particularly sensitive to changes of rhythmic patterns, we expect this strategy to yield insight (better: in-sound) into the resulting qualitative changes of behavior.

Examples S5 and S6 illustrates this behavior. It can be perceived directly when the coupling changes (after one third from beginning). This should be compared to the statistics, where at least 10000 interspike intervals are required before details of the correlations can be significantly shown. If correlations are evaluated from the complete time series where a change of parameter took place, the details are blurred due to this non-stationarity.

Sound Examples:

http://www.techfak.uni-bielefeld.de/~thermann/projects/index.html

S1: Audifications of time series from eq. (1) with D=0. Compression rates a) 2, b) 6, and c) 50.

S2: Amplitude modulation of time series from eq. (1) with D=0. Rates are a) 0.1, b) 0.2, and c) 1.

S3: Event-based rhythm with temporal compression 1 for a) D=0, and b) D=0.00125; and compression 5 for c) D=0, and d) D=0.00125.

S4: Timbre mapping at three different speeds (20, 60, and 240 sec) for D=0 (S4a-c), and D=0.00125 (S4d-f).

S5: Time series with time dependent coupling protocol: First third: D=0, second third: linear rise from D=0 to D=0.005, last third: D=0.005. a) Event-based rhythm sonification; b) as in a) but with additional timbre modulation; c) as in a) but with pitch deviation instead of timbre changes.

S6: Time series with time dependent coupling protocol: First third: D=0, second third: linear rise from D=0 to $D=D_{final}$, last third: $\boldsymbol{D=D_{final}}$. $\boldsymbol{D_{final}=0.00125}$ for S6a,b; and $D_{final}=0.005$ for S6d,e. Speed: 20 sec for S6a,c; and 30 sec for S6b,d.

5. DISCUSSION

Irregular spike sequences may appear to be random in univariate time series analysis and yet be correlated to some extent. The presence or absence of cross-correlation is not necessarily reflected in univariate methods of analysis and requires the application of bi- or multivariate methods. Methods based on bi-spectra and two-point cross-correlations have been developed for this purpose but we have focused on inter-spike intervals to highlight the usefulness of sonification in this context.

Topologically the extended FHN model in eq. (1) is related to other 3-variable FHN-type of equations like the Hindmarsh-Rose model [8]. The important feature at the given parameters was that spikes are simple (i.e., do not show the typical bursting pattern) and that their next neighbor interval distribution resembles a Poisson process independently of the value of the coupling constant D. This makes eq. (1) an ideal deterministic process where cross-correlations can be induced without affecting autocorrelations.

Two effects are found if coupling is introduced between two units. First, the number of spike coincidences increases. And second, there appear pronounced preferred short intervals of neighboring spikes in different units. If a good statistics is available both features can be quantified by means of the distribution as in Fig. 3. If such a statistics is not available, rhythmic sonification in a bivariate setting can be employed to search for the preferred intervals and other characteristic changes of the rhythmic pattern due to the coupling.

The important first step for a successful sonification of irregular rhythms is the choice of time scale. There is a clear optimum for the perception of temporal relationships as evidenced in our audifications and event-based sonifications. The optimum is in agreement with empricial findings on rhythm perception [9] but we have to take into account that our model time series with their broad-band Fourier spectra contain patterns on different time scales. Consequently, the choice of time scale depends on the task. For instance, considerable slowing down is required if the attention is not centered on the mean spiking rate but rather on the short inter-unit intervals of Fig. 3 (compare the event-based sonifications Sound S3).

In the analysis of long time series a constant timbre and pitch in the sonification tend to appear dull. However, this means that timbre and pitch offer auditory niches that can filled with additional information. In our case we exploit this possibility to offer information about "local properties", e.g. the recent past of the time series. Whereas for simple periodic rhythms this is merely an aesthetic improvement, for highly irregular rhythms as the one chosen the representation of the local properties improve the understanding of the dynamics (listen to Sound S5). This will be beneficial for the study of long time series where the changes of temporal patterns are complex and subtle as in the case of time-dependent weak cross-correlations (Sound S5 and S6).

Recently, a method of EEG analysis was proposed that is based on the conversion of the time series into spike trains [10,11]. Obviously the proposed sonification approach is equally applicable to the results of such an analysis. A rhythmic representation of results obtained from epileptic EEG data was presented in [12] but no emphasis was put on the inter-electrode spike intervals. We would like to stress that in the case of EEG analysis sonification might prove particularly valuable as the recordings are notoriously non-stationary and one is interested in transitions between the degree of cross-correlation, for example, when searching for precursors of epileptic seizures. For the special case of electric activity in the range from 1 to 12 Hertz (the delta, theta and alpha bands) the rhythmic representation can even be done in real-time as the intervals fall into the range of human rhythm perception [13].

6. ACKNOWLEDGEMENTS

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7. REFERENCES

[1] For the case of auditory nerve fibers see e.g. W.L. Gulick, G.A. Geschieder, R.D. Frisina, *Hearing - Physiological Acoustics, Neural Coding, and Psychoacoustics*. Oxford University Press, New York 1989.

[2] D. Oertel, R. Bal, S.M. Gardner, P.H. Smith, P.X. Joris, "Detection of synchrony in the activity auditory nerve fibers by octopus cells of the mammalian cochlear nucleus", Proc. Natl. Acad. Sci. **97**, 11773 (2000).

[3] S. Shamma, "On the role of space and time in auditory processing", Trends Cogn. Sci **5**, 340 (2001).

[4] O.E. Rössler, "An equation for continuous chaos", Phys. Lett. A **57**, 397 (1976).

[5] G. Baier, M. Müller and H. Ørsnes, "Excitable Spatio-Temporal Chaos in a Model of Glycolysis", J. Phys. Chem. B **106**, 3275 (2002).

[6] G. Baier, G.J. Escalera Santos, H. Perales, M. Rivera, M. Müller, R. Leder, P. Parmananda, "Self-exciting chaos as a dynamic model for irregular neural spiking", Phys. Rev. E **62**, 7579 (2000).

[7] http://www.techfak.uni-bielefeld.de/~thermann/projects/index.html

[8] J.L. Hindmarsh, R.M. Rose, Nature (London) **296**, 162 (1982); J.L. Hindmarsh, R.M. Rose, Proc. R. Soc. London, Ser. B, **221**, 87 (1984).

[9] P. Fraisse, "Rhythm and Tempo", in: D.Deutsch (ed.), *The Psychology of Music*, pp. 149-180.

[10] G. Baier, R. Leder, P. Parmananda, "Human Electroence-phalogram Induces Transient Coherence in Excitable Spatio-temporal Chaos", Phys. Rev. Lett. **84**, 4501 (2000).

[11] G. Baier and M. Müller, "The Nonlinear Dynamic Conversion of Analog Signals into Excitation Patterns", Phys. Rev. E **70**, 1 (2004).

[12] G. Baier and T. Hermann, "The Sonification of Rhythms in Human Electroencephalogram", Proceedings of ICAD 2004, July 6-9, Sydney (2004).

[13] T. Hinterberger and G. Baier, "POSER: Parametric Orchestral Sonification of EEG in Real-Time for the Self-Regulation of Brain States", IEEE Trans. Multimedia (2005), accepted.